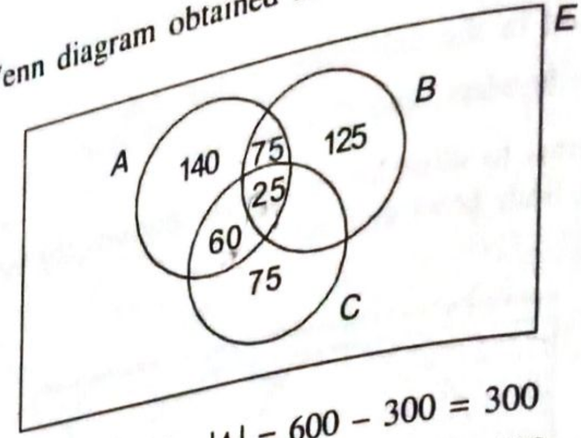
Solution

From the given data, the Venn diagram obtained is as follows:



(i) No. of female students $|A^c| = |E| - |A| = 600 - 300 = 300$

(ii) No. of students who are not bowlers $|B^c| = |E| - |B| = 600 - 225 = 375$

(iii) No. of students who are not batsmen $|C^c| = |E| - |C| = 600 - 160 = 440$

(iv) No. of female students who can bowl $|A^c \cap B| = 125$ (from the Venn diagram)

6.2.3 Partition and Covering

Partition

A *partition* on A is defined to be a set of non-empty subsets A_i, each of which is pairwise disjoint and whose union yields the original set A.

Partition on A indicated as $\Pi(A)$, is therefore

(i) $A_i \cap A_j = \varnothing$ for each pair $(i, j) \in I,\ i \neq j$ (6.17)

(ii) $\bigcup_{i \in I} A_i = A$

The members A_i of the partition are known as blocks (refer Fig. 6.8).

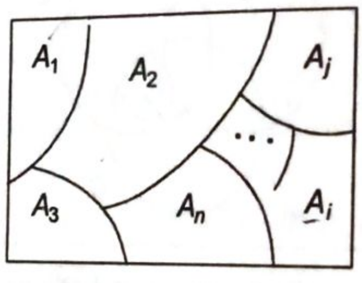


Fig. 6.8 Partition of set *A*.

Example

Given $A = \{a, b, c, d, e\}$, $A_1 = \{a, b\}$, $A_2 = \{c, d\}$ and $A_3 = \{e\}$, which gives

$$A_1 \cap A_2 = \varnothing,\ A_1 \cap A_3 = \varnothing,\ A_2 \cap A_3 = \varnothing$$

Also,

$$A_1 \cup A_2 \cup A_3 = A = \{a, b, c, d, e\}$$

Hence, $\{A_1, A_2, A_3\}$, is a partition on A.

Covering

A *covering* on A is defined to be a set of non-empty subsets A_i, whose union yields the original set A. The non-empty subsets need not be disjoint (Refer Fig. 6.9).

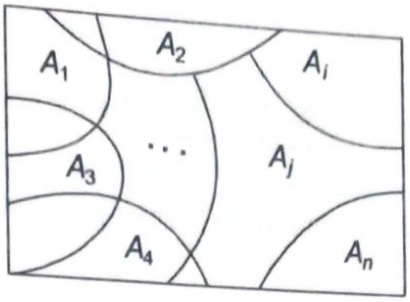


Fig. 6.9 Covering of set A.

Example

Given $A = \{a, b, c, d, e\}$, $A_1 = \{a, b\}$, $A_2 = \{b, c, d\}$, and $A_3 = \{d, e\}$. This gives

$$A_1 \cap A_2 = \{b\}$$

$$A_1 \cap A_3 = \varnothing$$

$$A_2 \cap A_3 = \{d\}$$

Also,
$$A_1 \cup A_2 \cup A_3 = \{a, b, c, d, e\} = A$$

Hence, $\{A_1, A_2, A_3\}$ is a covering on A.

Rule of Addition

Given a partition on A where A_i, $i = 1, 2, \ldots, n$ are its non-empty subsets then,

$$|A| = |\bigcup_{i=1}^{n} A_i| = \sum_{i=1}^{n} |A_i| \quad (6.18)$$

Example

Given $A = \{a, b, c, d, e\}$, $A_1 = \{a, b\}$, $A_2 = \{c, d\}$, $A_3 = \{e\}$, $|A| = 5$, and $\Sigma_{i=1}^{3} |A_i| = 2 + 2 + 1 = 5$

Rule of Inclusion and Exclusion

Rule of addition is not applicable on the covering of set A, especially if the subsets are not pairwise disjoint. In such a case, the rule of inclusion and exclusion is applied.

Example

Given A to be a covering of n sets $A_1, A_2, \ldots, A_n$,

for $n = 2$,
$$|A| = |A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2| \quad (6.19)$$

for $n = 3$,
$$|A| = |A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_2 \cap A_3| - |A_1 \cap A_3| + |A_1 \cap A_2 \cap A_3| \quad (6.20)$$

Generalizing,

$$|A| = |\bigcup_{i=1}^{n} A_i| = \sum_{i=1}^{n}|A_i| - \sum_{i=1}^{n}\sum_{\substack{j=1\\ i\neq j}}^{n}|A_i \cap A_j|$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{\substack{k=1\\ i\neq j\neq k}}^{n}|A_i \cap A_j \cap A_k| \ldots (-i)^{n+1}\left|\bigcap_{i=1}^{n} A_i\right|$$

Example 6.4

Given $|E| = 100$, where E indicates a set of students who have chosen subjects from diff streams in the computer science discipline, it is found that 32 study subjects chosen from Computer Networks (CN) stream, 20 from the Multimedia Technology (MMT) stream, and 45 f the Systems Software (SS) stream. Also, 15 study subjects from both CN and SS streams, 7 f both MMT and SS streams, and 30 do not study any subjects chosen from either of the th streams.

Find the number of students who study subjects belonging to all three streams.

Solution

Let A, B, C indicate students who study subjects chosen from CN, MMT, and SS strea respectively. The problem is to find $|A \cap B \cap C|$.

The no. of students who do not study any subject chosen from either of the th streams = 30.

i.e.

$$|A^c \cap B^c \cap C^c| = 30$$

$\Rightarrow$

$$|(A \cup B \cup C)^c| = 30 \quad \text{(using De Morgan's laws)}$$

$\Rightarrow$

$$|E| - |A \cup B \cup C| = 30$$

$\Rightarrow$

$$|A \cup B \cup C| = |E| - 30$$

$$= 100 - 30 = 70$$

From the principle of inclusion and exclusion,

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$\Rightarrow$

$$|A \cap B \cap C| = |A \cup B \cup C| - |A| - |B| - |C| + |A \cap B| + |B \cap C| + |A \cap C|$$

$$= 70 - 32 - 20 - 45 + 15 + 7 + 10$$

$$= 5$$

Hence, the no. of students who study subjects chosen from all the three streams is 5.

6.3 FUZZY SETS

Fuzzy sets support a flexible sense of membership of elements to a set. While in crisp set theory an element either belongs to or does not belong to a set, in fuzzy set theory many degrees of membership (between 0 and 1) are allowed. Thus, a membership function $\mu_A^{(x)}$ is associated with a fuzzy set $\tilde{A}$ such that the function maps every element of the universe of discourse X (or the reference set) to the interval [0, 1].

Formally, the mapping is written as $\mu_{\tilde{A}}(x) : X \to [0, 1]$

A *fuzzy set* is defined as follows:

If X is a universe of discourse and x is a particular element of X, then a fuzzy set A defined on X may be written as a collection of ordered pairs

$$A = \{(x, \mu_{\tilde{A}}(x)), x \in X\} \tag{6.23}$$

where each pair $(x, \mu_{\tilde{A}}(x))$ is called a singleton. In crisp sets, $\mu_{\tilde{A}}(x)$ is dropped.

An alternative definition which indicates a fuzzy set as a union of all $\mu_{\tilde{A}}(x)/x$ singletons is given by

$$A = \sum_{x_i \in X} \mu_{\tilde{A}}(x_i)/x_i \quad \text{in the discrete case} \tag{6.24}$$

and

$$A = \int_X \mu_{\tilde{A}}(x)/x \quad \text{in the continuous case} \tag{6.25}$$

Here, the summation and integration signs indicate the union of all $\mu_{\tilde{A}}(x)/x$ singletons.

Example

Let $X = \{g_1, g_2, g_3, g_4, g_5\}$ be the reference set of students. Let $\tilde{A}$ be the fuzzy set of "smart" students, where "smart" is a fuzzy linguistic term.

$$\tilde{A} = \{(g_1,0.4)\,(g_2,0.5)\,(g_3,1)\,(g_4,0.9)\,(g_5,0.8)\}$$

Here $\tilde{A}$ indicates that the smartness of g_1 is 0.4, g_2 is 0.5 and so on when graded over a scale of 0–1.

Though fuzzy sets model vagueness, it needs to be realized that the definition of the sets varies according to the context in which it is used. Thus, the fuzzy linguistic term "tall" could have one kind of fuzzy set while referring to the height of a building and another kind of fuzzy set while referring to the height of human beings.

6.3.1 Membership Function

The membership function values need not always be described by discrete values. Quite often, these turn out to be as described by a continuous function.

The fuzzy membership function for the fuzzy linguistic term "cool" relating to temperature may turn out to be as illustrated in Fig. 6.10.

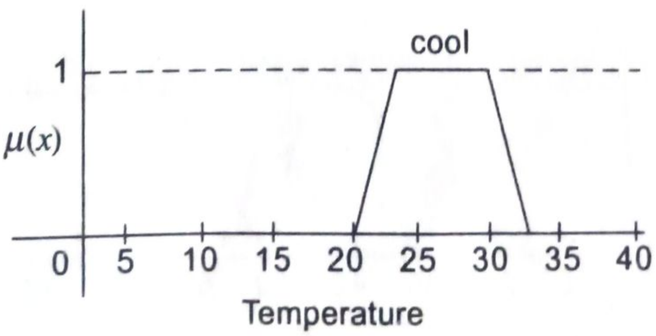


Fig. 6.10 Continuous membership function for "cool".

6.3.2 Basic Fuzzy Set Operations

Given X to be the universe of discourse and $\tilde{A}$ and $\tilde{B}$ to be fuzzy sets with $\mu_A(x)$ and $\mu_B(x)$ as their respective membership functions, the basic fuzzy set operations are as follows:

Union

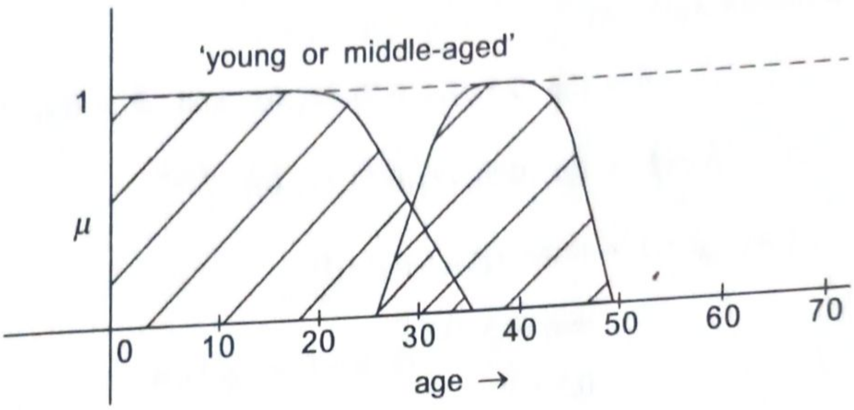
The union of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ is a new fuzzy set $\tilde{A} \cup \tilde{B}$ also on X with a membership function defined as

$$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x)) \tag{6.26}$$

Example

Let $\tilde{A}$ be the fuzzy set of young people and $\tilde{B}$ be the fuzzy set of middle-aged people as illustrated in Fig. 6.13. Now $\tilde{A} \cup \tilde{B}$, the fuzzy set of "young or middle-aged" will be given by

$\tilde{A} \cup \tilde{B}$:



In its discrete form, for x_1, x_2, x_3

if $A = \{(x_1, 0.5), (x_2, 0.7), (x_3, 0)\}$ and $\tilde{B} = \{(x_1, 0.8), (x_2, 0.2), (x_3, 1)\}$

$$\tilde{A} \cup \tilde{B} = \{(x_1, 0.8), (x_2, 0.7), (x_3, 1)\}$$

since,

$$\mu_{\tilde{A} \cup \tilde{B}}(x_1) = \max(\mu_{\tilde{A}}(x_1), \mu_{\tilde{B}}(x_1))$$

$$= \max\ (0.5, 0.8)$$

$$= 0.8$$

$$\mu_{\tilde{A} \cup \tilde{B}}(x_2) = \max(\mu_{\tilde{A}}(x_2), \mu_{\tilde{B}}(x_2)) = \max(0.2, 0.7) = 0.7$$

$$\mu_{\tilde{A} \cup \tilde{B}}(x_3) = \max(\mu_{\tilde{A}}(x_3), \mu_{\tilde{B}}(x_3)) = \max(0, 1) = 1$$

Intersection

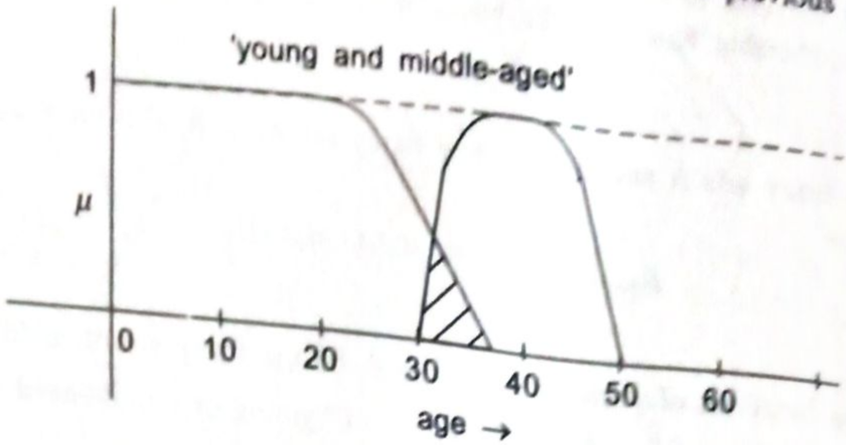
The intersection of fuzzy sets $\tilde{A}$ and $\tilde{B}$ is a new fuzzy set $\tilde{A} \cap \tilde{B}$ with membership function defined as

$$\mu_{\tilde{A} \cap \tilde{B}}(x) = \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)) \tag{6.27}$$

Example

For $\tilde{A}$ and $\tilde{B}$ defined as "young" and "middle-aged" as illustrated in previous examples.

$\tilde{A} \cap \tilde{B}$:



In its discrete form, for x_1, x_2, x_3

if

$$\tilde{A} = \{(x_1,0.5),(x_2,0.7),(x_3,0)\} \text{ and } \tilde{B} = \{(x_1,0.8),(x_2,0.2),(x_3,1)\}$$

$$\tilde{A} \cap \tilde{B} = \{(x_1,0.5),(x_2,0.2),(x_3,0)\}$$

since,

$$\mu_{\tilde{A}\cap\tilde{B}}(x_1) = \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{B}}(x_1))$$
$$= \min(0.5, 0.8)$$
$$= 0.5$$

$$\mu_{\tilde{A}\cap\tilde{B}}(x_2) = \min(\mu_{\tilde{A}}(x_2), \mu_{\tilde{B}}(x_2))$$
$$= \min(0.7, 0.2)$$
$$= 0.2$$

$$\mu_{\tilde{A}\cap\tilde{B}}(x_3) = \min(\mu_{\tilde{A}}(x_3), \mu_{\tilde{B}}(x_3))$$
$$= \min(0,1)$$
$$= 0$$

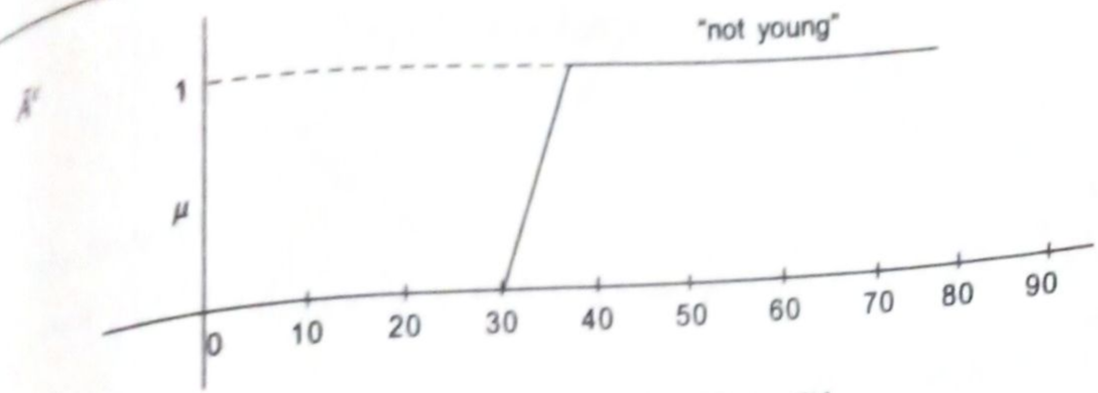
Complement

The complement of a fuzzy set $\tilde{A}$ is a new fuzzy set $\tilde{A}$ with a membership function

$$\mu_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) \tag{6.28}$$

Example

For the fuzzy set $\tilde{A}$ defined as "young" the complement "not young" is given by $\tilde{A}^c$. In its discrete form, for x_1, x_2, and x_3



$$\tilde{A} = \{(x_1, 0.5)\ (x_2, 0.7)\ (x_3, 0)\}$$
$$\tilde{A}^c = \{(x_1, 0.5)\ (x_2, 0.3)\ (x_3, 1)\}$$

$$\begin{aligned}\mu_{\tilde{A}^c}(x_1) &= 1 - \mu_{\tilde{A}}(x_1)\\ &= 1 - 0.5\\ &= 0.5\end{aligned}$$

$$\begin{aligned}\mu_{\tilde{A}^c}(x_2) &= 1 - \mu_{\tilde{A}}(x_2)\\ &= 1 - 0.7\\ &= 0.3\end{aligned}$$

$$\begin{aligned}\mu_{\tilde{A}^c}(x_3) &= 1 - \mu_{\tilde{A}}(x_3)\\ &= 1 - 0\\ &= 1\end{aligned}$$

Other operations are,

Product of two fuzzy sets

The product of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ is a new fuzzy set $\tilde{A} \cdot \tilde{B}$ whose membership function is defined as

$$\mu_{\tilde{A}\cdot\tilde{B}}(x) = \mu_{\tilde{A}}(x)\,\mu_{\tilde{B}}(x) \tag{6.29}$$

Example

$$\tilde{A} = \{(x_1, 0.2), (x_2, 0.8), (x_3, 0.4)\}$$
$$\tilde{B} = \{(x_1, 0.4)\ (x_2, 0), (x_3, 0.1)\}$$
$$\tilde{A} \cdot \tilde{B} = \{(x_1, 0.08)\ (x_2, 0)\ (x_3, 0.04)\}$$

Since

$$\begin{aligned}\mu_{\tilde{A}\cdot\tilde{B}}(x_1) &= \mu_{\tilde{A}}(x_1) \cdot \mu_{\tilde{A}}(x_2)\\ &= 0.2 \cdot 0.4 = 0.08\end{aligned}$$

$$\begin{aligned}\mu_{\tilde{A}\cdot\tilde{B}}(x_2) &= \mu_{\tilde{A}}(x_2) \cdot \mu_{\tilde{A}}(x_2)\\ &= 0.8 \cdot 0 = 0\end{aligned}$$

$$\mu_{\tilde{A}\cdot\tilde{B}}(x_3) = \mu_{\tilde{A}}(x_3)\cdot\mu_{\tilde{A}}(x_3)$$
$$= 0.4 \cdot 0.1$$
$$= 0.04$$

Equality

Two fuzzy sets $\tilde{A}$ and $\tilde{B}$ are said to be equal ($\tilde{A} = \tilde{B}$) if $\mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x)$ (6.30)

Example

$$\tilde{A} = \{(x_1, 0.2)(x_2, 0.8)\}$$
$$\tilde{B} = \{(x_1, 0.6)(x_2, 0.8)\}$$
$$\tilde{C} = \{(x_1, 0.2)(x_2, 0.8)\}$$
$$\tilde{A} \neq \tilde{B}$$

since

$$\mu_{\tilde{A}}(x_1) \neq \mu_{\tilde{B}}(x_1) \text{ although}$$
$$\mu_{\tilde{A}}(x_2) = \mu_{\tilde{B}}(x_2)$$

but

$$\tilde{A} = \tilde{C}$$

since

$$\mu_{\tilde{A}}(x_1) = \mu_{\tilde{C}}(x_1) = 0.2$$

and

$$\mu_{\tilde{A}}(x_2) = \mu_{\tilde{C}}(x_2) = 0.8$$

Product of a fuzzy set with a crisp number

Multiplying a fuzzy set $\tilde{A}$ by a crisp number a results in a new fuzzy set product a. $\tilde{A}$ with the membership function

$$\mu_{a\cdot\tilde{A}}(x) = a\cdot\mu_{\tilde{A}}(x) \quad (6.31)$$

Example

For

$$\tilde{A} = \{(x_1, 0.4), (x_2, 0.6), (x_3, 0.8)\}$$
$$a = 0.3$$
$$a\cdot\tilde{A} = \{(x_1, 0.12), (x_2, 0.18), (x_3, 0.24)\}$$

since,

$$\mu_{a\cdot\tilde{A}}(x_1) = a\cdot\mu_{\tilde{A}}(x_1)$$
$$= 0.3 \cdot 0.4$$
$$= 0.12$$

$$\mu_{a\cdot\tilde{A}}(x_2) = a\cdot\mu_{\tilde{A}}(x_2)$$
$$= 0.3 \cdot 0.6$$
$$= 0.18$$

$$\mu_{a\cdot\tilde{A}}(x_3) = a\cdot\mu_{\tilde{A}}(x_3)$$
$$= 0.3 \cdot 0.8$$
$$= 0.24$$

Power of a fuzzy set

The α power of a fuzzy set $\tilde{A}$ is a new fuzzy set A^{α} whose membership function is given by

$$\mu_{A^{\alpha}}(x) = (\mu_{\tilde{A}}(x))^{\alpha} \tag{6.32}$$

Raising a fuzzy set to its second power is called *Concentration* (CON) and taking the square root is called *Dilation* (DIL).

Example

$$\tilde{A} = \{(x_1,0.4),(x_2,0.2),(x_3,0.7)\}$$
$$\alpha = 2$$

For

$$\mu_{\tilde{A}^2}(x) = (\mu_{\tilde{A}}(x))^2$$
$$(\tilde{A})^2 = \{(x_1,0.16),(x_2,0.04),(x_3,0.49)\}$$

Hence,

Since

$$\mu_{\tilde{A}^2}(x_1) = (\mu_{\tilde{A}}(x_1))^2 = (0.4)^2 = 0.16$$
$$\mu_{\tilde{A}^2}(x_2) = (\mu_{\tilde{A}}(x_2))^2 = (0.2)^2 = 0.04$$
$$\mu_{\tilde{A}^2}(x_3) = (\mu_{\tilde{A}}(x_3))^2 = (0.7)^2 = 0.49$$

Difference

The difference of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ is a new fuzzy set $\tilde{A}-\tilde{B}$ defined as

$$\tilde{A}-\tilde{B} = (\tilde{A}\cap\tilde{B}^c) \tag{6.33}$$

Example

$$\tilde{A} = \{(x_1,0.2),(x_2,0.5),(x_3,0.6)\};\ \tilde{B} = \{(x_1,0.1),(x_2,0.4),(x_3,0.5)\}$$
$$\tilde{B}^c = \{(x_1,0.9),(x_2,0.6),(x_3,0.5)\}$$
$$\tilde{A}-\tilde{B} = \tilde{A}\cap\tilde{B}^c$$
$$= \{(x_1,0.2)(x_2,0.5)(x_3,0.5)\}$$

Disjunctive sum

The disjunctive sum of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ is a new fuzzy set $\tilde{A}\oplus\tilde{B}$ defined as

$$\tilde{A}\oplus\tilde{B} = (\tilde{A}^c\cap\tilde{B})\cup(\tilde{A}\cap\tilde{B}^c) \tag{6.34}$$

Example

$$\tilde{A} = \{(x_1, 0.4)(x_2, 0.8)(x_3, 0.6)\}$$

$$\tilde{B} = \{(x_1, 0.2)(x_2, 0.6)(x_3, 0.9)\}$$

Now,

$$\tilde{A}^c = \{(x_1, 0.6)(x_2, 0.2)(x_3, 0.4)\}$$

$$\tilde{B}^c = \{(x_1, 0.8)(x_2, 0.4)(x_3, 0.1)\}$$

$$\tilde{A}^c \cap \tilde{B} = \{(x_1, 0.2)(x_2, 0.2)(x_3, 0.4)\}$$

$$\tilde{A} \cap \tilde{B}^c = \{(x_1, 0.4)(x_2, 0.4)(x_3, 0.1)\}$$

$$\tilde{A} \oplus \tilde{B} = \{(x_1, 0.4)(x_2, 0.4)(x_3, 0.4)\}$$

6.3.3 Properties of Fuzzy Sets

Fuzzy sets follow some of the properties satisfied by crisp sets. In fact, crisp sets can be tho of as special instances of fuzzy sets. Any fuzzy set $\tilde{A}$ is a subset of the reference set X. Also, membership of any element belonging to the null set $\varnothing$ is 0 and the membership of any ele belonging to the reference set is 1.

The properties satisfied by fuzzy sets are

Commutativity:

$$\tilde{A} \cup \tilde{B} = \tilde{B} \cup \tilde{A}$$

$$\tilde{A} \cap \tilde{B} = \tilde{B} \cap \tilde{A}$$

Associativity:

$$\tilde{A} \cup (\tilde{B} \cup \tilde{C}) = (\tilde{A} \cup \tilde{B}) \cup \tilde{C}$$

$$\tilde{A} \cap (\tilde{B} \cap \tilde{C}) = (\tilde{A} \cap \tilde{B}) \cap \tilde{C}$$

Distributivity:

$$\tilde{A} \cup (\tilde{B} \cap \tilde{C}) = (\tilde{A} \cup \tilde{B}) \cap (\tilde{A} \cup \tilde{C})$$

$$\tilde{A} \cap (\tilde{B} \cup \tilde{C}) = (\tilde{A} \cap \tilde{B}) \cup (\tilde{A} \cap \tilde{C})$$

Idempotence:

$$\tilde{A} \cup \tilde{A} = \tilde{A}$$

$$\tilde{A} \cap \tilde{A} = \tilde{A}$$

Identity:

$$\tilde{A} \cup \varnothing = \tilde{A}$$

$$\tilde{A} \cup X = \tilde{A}$$

$$\tilde{A} \cap \varnothing = \varnothing$$

$$\tilde{A} \cup X = X$$

Transitivity: If $\tilde{A} \subseteq \tilde{B} \subseteq \tilde{C}$, then $\tilde{A} \subseteq \tilde{C}$

Involution:

$$(\tilde{A}^c)^c = \tilde{A}$$

De Morgan's laws:

$$(\tilde{A} \cap \tilde{B})^c = (\tilde{A}^c \cup \tilde{B}^c)$$

$$(\tilde{A} \cup \tilde{B})^c = (\tilde{A}^c \cap \tilde{B}^c)$$

Since fuzzy sets can overlap, the laws of excluded middle do not hold good. Thus,

$$\tilde{A} \cup \tilde{A}^c \neq X \tag{6.43}$$

$$\tilde{A} \cap \tilde{A}^c \neq \varnothing \tag{6.44}$$

Example 6.5

The task is to recognize English alphabetical characters (F, E, X, Y, I, T) in an image processing system.

Define two fuzzy sets $\tilde{I}$ and $\tilde{F}$ to represent the identification of characters I and F.

$$\tilde{I} = \{(F, 0.4), (E, 0.3), (X, 0.1), (Y, 0.1), (I, 0.9), (T, 0.8)\}$$

$$\tilde{F} = \{(F, 0.99), (E, 0.8), (X, 0.1), (Y, 0.2), (I, 0.5), (T, 0.5)\}$$

Find the following.

(a) (i) $\tilde{I} \cup \tilde{F}$ (ii) $(\tilde{I} - \tilde{F})$ (iii) $\tilde{F} \cup \tilde{F}^c$

(b) Verify De Morgan's Law, $(\tilde{I} \cup \tilde{F})^c = \tilde{I}^c \cap \tilde{F}^c$

Solution

(a) (i) $$\tilde{I} \cup \tilde{F} = \{(F,0.99), (E,0.8), (X,0.1), (Y,0.2), (I,0.9), (T,0.8)\}$$

(ii) $$\tilde{I} - \tilde{F} = (\tilde{I} \cap \tilde{F}^c)$$
$$= \{(F,0.01), (E,0.2), (X,0.1), (Y,0.1), (I,0.5), (T,0.5)\}$$

(iii) $$\tilde{F} \cup \tilde{F}^c = \{(F,0.99), (E,0.8), (X,0.9), (Y,0.8), (I,0.5), (T,0.5)\}$$

(b) De Morgan's Law

$$(\tilde{I} \cup \tilde{F})^c = \tilde{I}^c \cap \tilde{F}^c$$

$$\tilde{I} \cup \tilde{F} = \{(F,0.99), (E,0.8), (X,0.1), (Y,0.2), (I,0.9), (T,0.8)\}$$

$$(\tilde{I} \cup \tilde{F})^c = \{(F,0.01), (E,0.2), (X,0.9), (Y,0.8), (I,0.1), (T,0.2)\}$$

$$\tilde{I}^c = \{(F,0.6), (E,0.7), (X,0.9), (Y,0.9), (I,0.1), (T,0.2)\}$$

$$\tilde{F}^c = \{(F,0.01), (E,0.2), (X,0.9), (Y,0.8), (I,0.5), (T,0.5)\}$$

and

$$\tilde{I}^c \cap \tilde{F}^c = \{(F,0.01), (E,0.2), (X,0.9), (Y,0.8), (I,0.1), (T,0.2)\}$$

Hence, $$(\tilde{I} \cup \tilde{F})^c = \tilde{I}^c \cap \tilde{F}^c$$